AUGMENTED REALITY: A SPACE FOR THE UNDERSTANDING OF MULTI-VARIATE CALCULUS

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Abstract

Technology combined with adequate pedagogical methodologies can give birth to new ways to educate. Augmented Reality enables the integration of 3-D virtual objects within a 3-D real environment in real time and has been used in a number of medical, manufacturing, visualization, path-planning, entertainment and military applications.

We have created an Augmented Reality application that allows an instructor and a student to collaboratively visualize and interact with mathematical concepts represented by virtual objects that appear to be mixed with reality. The virtual objects appear to float above a pattern in front of their eyes. As they move the physical object that contains the pattern, the virtual objects move accordingly, allowing the instructor and the student to see the illustrated mathematical concepts from any angle they want.

In this paper, we present the results obtained from two groups of students, one experimental group (tutored using the AR application), and one control group (tutored using traditional windowed applications). The tutoring was based on the Teaching for Understanding framework, that encourages students to apply acquired concepts and relate them to every-day life.

The Teaching for Understanding framework was also used to assess the level of understanding of the students in both groups. A panel of experts graded the students´ levels of understanding (naive, novice, apprentice or master) in the different dimensions of understanding (method, purpose, contents and form).

The results show that this approach for teaching Calculus improves the students´ understanding and provides a basis for further exploration of the application of Augmented Reality in the teaching of mathematics.

Keywords
Augmented Reality, Teaching for Understanding, multi-variate calculus.

1. INTRODUCTION

Education in all areas of knowledge and in particular the teachings of mathematics benefits from the technological developments with the creation of new spaces in which professors and students can have new forms to visualize the subject. The space that is created with the help of augmented reality allows the user to perceive the real world along with virtual objects imposed over the real world, completing the reality instead of replacing it and permitting the interaction of the users with the real objects and the virtual objects in real time. 3D images created with augmented reality facilitate the visualization of $z= f(x,y)$ surfaces that are studied in Multi-Variate Calculus.

Education is called to look for strategies of change and renovation in response to a society that is in constant transformation. It is impossible to think that institutions of education are far from the permanent innovations that are happening in the environment. One possible strategy is to generate a process of constant reflection that can help establish plans of action. This will articulate the new technology between teaching and learning to give the students the abilities that will allow them to interpret the concepts that are being studied in diverse situations in the environment.

The Virtual Reality lab at Eafit University has been working on the "Augmented Reality for Teaching Multi-Variate Calculus" project, which in the first stage aims at developing a tool of Augmented Reality to facilitate

the teachings of Multi-Variate Calculus, having as its base the Teaching for Understanding which was proposed by David Perkins, Howard Gardner, and the team of Project Zero team at Harvard University[13]. This way of teaching allows to design and evaluate units in a flexible manner. It takes in to account a powerful theory that integrates four components and four dimensions . Through the teachings for understanding we aim at determining the characteristics of comprehension and the type of educative acts that favour it.

1.1 Teaching for Understanding framework

The teaching for understanding has four dimensions: (1) The *contents* that are directly related with the curriculum, (2) the *methods* which are the forms that new knowledge is constructed, (3) the *practice*, which is the specific manner in which the practice functions. (4) the *form* of communication, which is related to language in the specific area of knowledge. In the transition from a practice to teaching in order to create an understanding, it is necessary to take into account four components: (1) The *goals of understanding*, what level of understanding will reach the students at the end of the process of teaching and learning. (2) the *genarative topics*, what motivates the students to learn a new concept? (3) The *performances of that understanding*, what activities help increase and demonstrate his or her level of understanding? (4) The *Ongoing Assessment*, how to evaluate the progress of the understanding of the concept?

Understanding is developed within the student as it stimulates "the capacity to generate with a topic a variety of experiences that stimulate the mind. For instance, explain, demonstrate and give examples, generalize, establish analogies and once again present the topic in a new manner. As a result, understanding transcends the knowledge itself. When you understand something you can come up with various actions that demonstrate that it is capable of incorporating the knowledge and utilizing it in an innovative manner[4].

1.2 Augmented Reality in teaching and understanding mathematics

Augmented Reality is a set variation of virtual environments (VE), which is also commonly known as Virtual Reality. Virtual Reality submerges the user into a synthetic, computer generated environment. While the user is immersed in virtual reality, he or she cannot see the real world that surrounds them. In contrast, Augmented Reality permits the user to see the real world with virtual objects that are superimposed over the real world or composed along with it [2].

In the investigation, "Augmented Reality in the teachings of Mathematics", a specialized software was designed which allowed a professor and a student to interact and visualize the 3D surfaces generated by the computer through a video camera and virtual reality glasses, permitting the comparison between the virtual object and real life objects[1], see Fig. 1

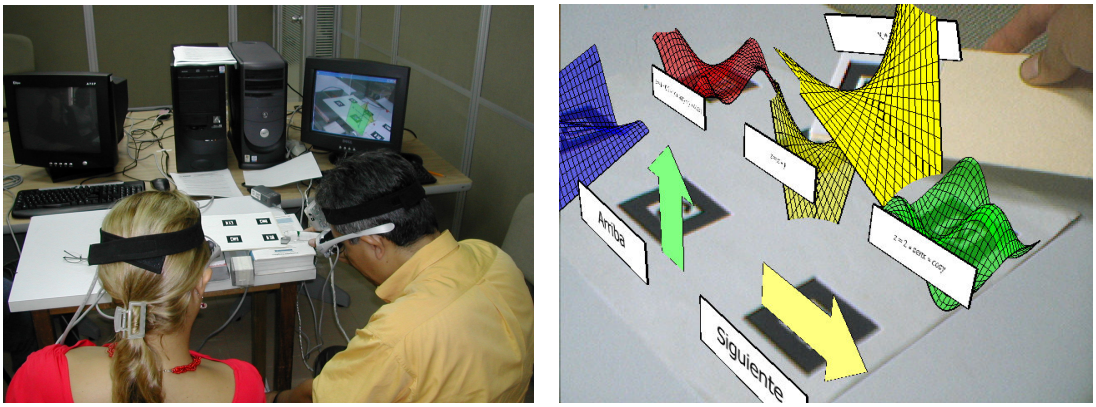


Fig. 1: Professor and student interacting with the Augmented Reality tool with previously selected surfaces from a predefined menu.

The Teaching for Understanding framework, used as the pedagogical approach in the project, permitted the design of certain types of questions that were given to the students during the tutorials, like the type of graphics they interacted with when using the Augmented Reality tool.

2. DEVELOPMENT OF THE EXPERIENCE AND RESULTS OBTAINED

To accomplish the experiment, two random groups of students were taken from different branches of the engineering field: one of them was an experimental group and the other was the control group. To develop the different programmed activities throughout the Multi-Variate Calculus course, each group was divided in small subgroups of two or three students, with a total of six subgroups from each class. This came to a total of fourteen students in each one. Throughout the course, each subgroup of the experimental group attended a weekly tutorial for half an hour with Augmented Reality as a tool. All the students involved in the experiment developed the activities designed for the development of the course in the TFU framework. In order to guarantee that all of the students received the same instructions and teachings, the same professor was assigned to all of them.

2.1 The Multi-Variate Calculus course and Teaching for Understanding

In order to determine the main topics of the course, several meetings were held among several professors. This permitted them to construct a conceptual map that served as a fundamental basis to determine the four dimensions and the four components of the Teachings for Understanding framework.

Because calculus is a language utilized by other branches of science to communicate and interpret mathematical results, with an adequate teaching design it can motivate the students so they can practice it in their future professions and their environment. From this perspective, the goals of understanding will be motivated by questions that do not have definite answers. On the contrary, as the course progresses ,as well as new concepts are acquired, the possible results will be qualified. The goals of understanding presented to the students were the following: How do technological advances help them improve the understanding of calculus? How does calculus influence our lives and help modify the world? What things with which I relate to daily are a product of calculus? Can calculus help construct the future?

Taking into account that the students involved in the experiment belonged to different branches within the engineering field and that the same concept of calculus can represent diverse concepts from other areas of knowledge, it was determined that calculus was the main generative topic. Because it is such an expansive topic, and in order to orient the understanding of concepts, it was divided into eight parts one of which was the following: Given a certain surface, the student will describe it in Cartesian coordinates, cylindrical and spherical taking into account the different parameters for each of those systems. The limits of integration will be interchanged in each of these systems of coordinates to calculate diverse volumes.

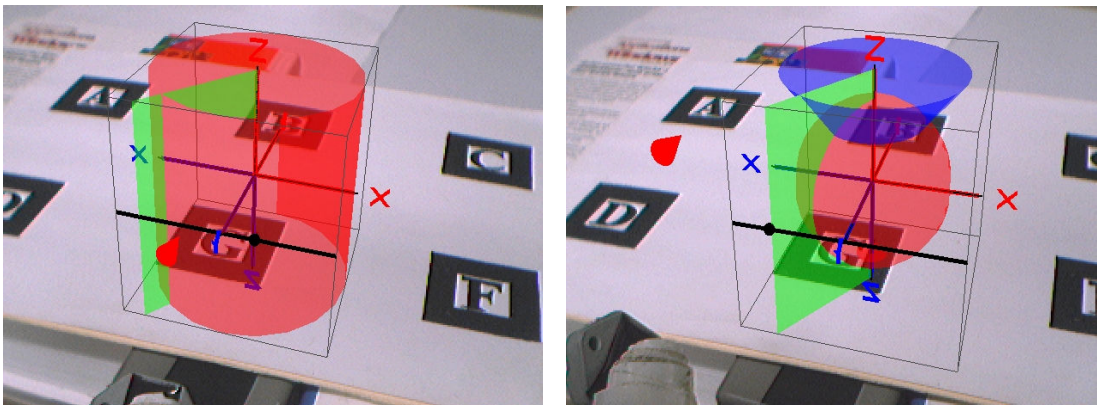


Fig. 2. Graphics generated with the Augmented Reality tool with which the students interacted in the study of cylindrical and spherical coordinates.

To orient the results of the understanding with which the students applied the concepts they previously learned the steps they undertook were taken into account. They are a creative manifestation of the understanding and the mental domain of this new knowledge acquired that can be utilized to provide new ideas and to solve new problems. This experiment was divided in three stages: The *preliminary results*, in which different manifestations of calculus in daily life were observed; *guided investigation*, in which depending on the interests of the students, a design of a concrete object with which they could apply and explain various concepts studied throughout the course; finally, the *synthesis*, which culminates the socialization of the concrete objects designed with calculus professors, peers and experts in each of those works.

For the design of each of these works, the students based their own work on graphic surface of the form $z = f(x,y)$. The designs formed by the experimental group were the following: various lamps, a seat used to mount a horse, models of buildings ,knives, living room furniture and a rack. The works done by the control group were the following: Software that explained sound, tents utilized for camping, musical chairs, a telescope, and antennae. With the use of these objects, each student had to explain the type of surfaces involved, the concepts of calculus applied and the possibility to bring about its construction.

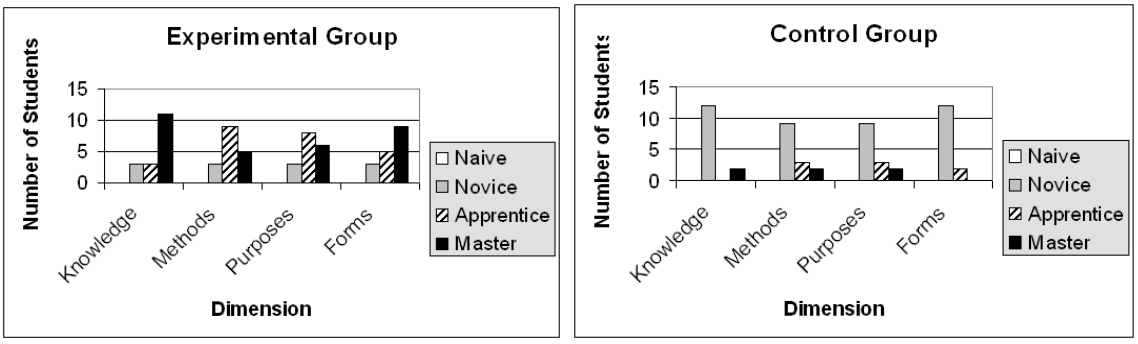
2.2 The Ongoing Assesment

For the dimension of ongoing assessment, the students depended on the knowledge of various professors and other experts in the field. In addition, students in the experimental group attended the tutorials regularly with the use of the Augmented Reality tool.

Also, the evaluation required by the university, brought about a type of qualitative evaluation that was in tune with Teaching for Understanding. This was based upon on three important aspects; Presentation, the utilization of the concepts in calculus that deal with Multi-Variate and problem solving strategies. Each of these aspects were organized in tables that involved different manifestations of the understanding. In the last presentation, the professors, as well as the experts, assigned a qualitative grade that could be either *naive*, *novice*, *apprentice* or an *expert*. One pertinent aspect is that the experts that took part in the final evaluation did not know which students were in either the experimental group or the control group. When looking at the results, the students that were in the experimental group fell between the apprentices and the experts , while those in the control group fell primarily in the categories of novices and apprentices. Only the students that developed the software itself were considered experts.

3. RESULTS

The following graphics show the experts' assessment of the students level of understanding.



According to the experts, the experimental group showed in their final assignment a good conceptualization and concept application, good fundamental basis and ability to argue about multi-variate calculus concepts. The visualization using the Augmented Reality tool influenced the making of the final assignments. Some aspects to highlight about the experimental group are:

- A deeper understanding of the concepts and a better ability to apply them.
- In the beginning of the process, the students were concerned only with the final outcome and knowing whether it was "right" or "wrong". When presenting their projects, they showed independence, wittiness and confidence in their own learning process.
- The novelty of the Augmented Reality tool and working in their final assignments in conditions different from traditional lessons, encouraged independence from the teacher and resourcefulness.
- All the students showed interest in the role technology plays to support learning, in particular the Augmented Reality tool.

In the beginning, the control group were motivated and showed great interest to use different software programs to solve calculus exercises, but they did not achieve a good integration between the studied concepts and their final assignments. The most relevant characteristics of the control group were:

- There was a noticeable low commitment by the students to their own learning process.

- It was not easy for them to apply appropriate studying habits and, above all, they were not able to tell which methods were more adequate to use in their final assignments.
- Heteronomy was present as students did not pose any questions or ideas for the final assignment, and displayed poor initiative.
- Poor understanding of the studied concepts and difficulty to apply them to their final assignment, major and professional life.

4. CONCLUSIONS

The combination of the Teaching for Understanding pedagogy, the Augmented Reality tool and the students' practical work promoted the students' insight and self-esteem to face mathematical learning processes. This can be concluded from the questionnaires. Some students' comments follow:

- "The topic we saw today impressed me, because I could not have imagined that in saddle point the tangent plane had the property of being both over and under a mathematical surface. I think I would not have understood that in a traditional classroom".
- "I felt I was solving so many doubts and that I will never forget what I was watching, because I not only solved my doubt, but I understood".
- "This is a different experience. This methodology goes beyond exercises and texts to interact with calculus daily".

The integration between pedagogy and technology in teaching abstract mathematical concepts was remarkable, as it enabled students to relate those concepts to everyday life.

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