COLLABORATIVE AND DISTRIBUTED AUGMENTED REALITY IN TEACHING MULTI-VARIATE CALCULUS

Carlos Orozco
Computer Science Department
Eafit University
A.A. 3300
Medellín, Collombia
email: corozcop@eafit.edu.co

Pedro Esteban
Basic Sciences Department
Eafit University
A. A. 3300
Medellín, Colombia
email: pesteban@eafit.edu.co

Helmuth Trefftz
Department of Computer Science
Eafit University
A. A. 3300
Medellín, Colombia
email: htrefftz@eafit.edu.co

ABSTRACT

This article presents the first results of using an Augmented Reality (AR) tool, designed to support tutoring sessions in multi-variate calculus. The tool is used either in a face-to-face setting in which the instructor and the students are collocated or in a distance setting, in which the instructor and students are physically in remote places. The tool was used with two groups of students of Differential Calculus. The students had not been exposed to the concept of equations involving 3 variables and the corresponding surfaces in space. The experience explored how students generalized 2D graphics and equations with their 3D surfaces counterparts with the help of the tool.

KEY WORDS
Augmented Reality, visualization, function transformation, surfaces

1. Introduction

Learning experiences can be enhanced with the use of computer based tools, allowing students for a faster and deeper understanding of topics at hand. Students also seem to establish more and better relations of the studied concepts with other concepts and with real life objects.

An Augmented Reality tool has been developed at Eafit University to be used in the teaching of calculus, physics, chemistry, biology and civil engineering structures, among others. The tool allows an instructor and a student to observe and manipulate a shared virtual object, while being able to talk to each other and observe any other aspect of reality (opposed to immersive VR systems in which users are disconnected temporarily from reality while immersed in a virtual world).

A distance collaboration module has been added to the tool, allowing the instructor and the student to conduct a tutoring session, while being physically located in different places. The instructor and the student use pointing devices that the application tracks. The position and orientation of the device is sent over the network to the remote application, which displays a 3D arrow representing the pointing device of the remote user.

This paper describes the results of using the tool with a group of students of Differential Calculus. The students had not been exposed to equations with 3 unknowns, nor to their corresponding 3D surfaces. The idea of the experience was to identify how the tool could enhance the understanding of these new concepts by thep students.

The rest of the paper is organized as follows: Section 2 describes several projects this was based upon. The aplication is described in section 3. How the tool is used to visualize functions is the topic of section 4. Section 5 describes the design and implementation of the learning experience. The results are described in section 6. And, finally, sections 7 and 8 present the future work and conclusions.

2. Related Work

Augmented Reality has been used as a collaborative tool in many different ways. One of the first systems developed for creating collaborative environments using AR was Studierstube[1], where users were able to customize their own visualization, and collaborate with other users in the manipulation of virtual structures overlapped over real patterns. As a result of the Studierstube project [2], the users showed an increase of their understanding about the structures, also the interaction between users increased the participation through discussions, helping and improving the learning process.

In remote collaboration, the effort has been focused, in most cases, on Video Conferencing systems ([3]). In these systems, a video stream overlaps a real surface. In order to simulate a real conference room, each surface represents one person seated in one place. Every user is captured by a camera and this video is transmitted to the others and displayed over patterns in an AR environment. It creates a closer approximation to what a real meeting room is, but the integration between the real and the virtual environment is just a 2D integration.

Trying to allow a better space integration, the National University of Singapore ([4]) developed a system in which several cameras are used for capturing the remote users from different views in order to display an exact perspective based on the observer's real world location. This created a better 3D immersion between the real environment and the video displayed to the remote users.

3. System Properties

The system was designed to improve a student's comprehension of surfaces in a three variable equation. To achieve this, the application was divided in an instructor-side interface and in a student-side one. The former, allows the instructor to type in equations in his own control panel with the possibility of altering the parameters that define the visualization. This 3D surface is shared by the instructor and the student as described in Figure 1. The instructor-side application is called *server* and the student-side one is called *client*. The *server* has a panel that provides the instructor with real time control of the visualization. Using this panel, the instructor can change the evaluation range, the evaluation variation rate and the drawing scale. The *client* application shows to the students the same surface that is being displayed in the *server*, and it draws all the changes that the teacher does.

Figura 1. Instructor and student interact locally.

4. Function Visualization

In a first phase, the tool was used in a one-semester multi-variate calculus course. The tool was used to visualize surfaces of the from $z = f(x, y)$, as well as related concepts, such as: directional derivative, double and triple integrals, surface area, density, center of mass, tangent lines to various trajectories in the surfaces and tangent planes, among others ([8]).

In the current phase, two hypothesis are being investigated: (i) Can the use of the tool increase the understanding of the surface concept by students who have not been exposed to a multi-variate calculus course? and (ii) Can the use of the tool help students in extrapolating 2D concepts (such as translation, scaling and reflection) to surfaces in 3D? We are also performing initial tests in which the student and the instructor are physically in separate places.

Visualization plays a very important role in the teaching and learning of mathematical concepts. According to Zimmerman and Cunningham, "If Mathematics is the science of patterns, trying to find the most effective visualization techniques of these patterns and using visualization creatively as a tool for understanding is natural. This is the essence of mathematical visualization" ([7]). This concepts apply particularly well to Calculus. When learning Calculus, it is fundamental for students to recognize the concepts of formation and deduction, not only symbolically, but also in a graphical manner. It is also fundamental for the students to relate calculus concepts to their surroundings. The AR tool allows students to *manipulate* virtual objects as if they were real objects (by either moving the pattern they cause the objects to move, or by moving their heads they move the point of view in a very intuitive manner). This natural interface allows for the involvement of several senses in the learning of the concept at hand.

5. Design and deployment of the learning experience

While designing the experience, functions of the form $y = f(x)$, such as $y = x$, $y = x^2$, $y = \ln(x)$ and $y = e^x$ were considered. These functions are the most commonly used in Differential Calculus courses. Students had already been exposed to concepts such as translation, scaling and reflections of these functions, and they had drawn these functions on paper. Students had not been exposed to graphics calculators or visualization math programs with symbolic or graphic capabilities.

A section was chosen from a Differential Calculus course. The group was further divided into two subgroups. One group received face-to-face tutoring and the other remote tutoring, both using the AR tool.

When starting the experience, each group received basic treinning on the use of the tool (see figure 2). The different hardware and software components were used, as well as their use. The objective was to provide conditions for the tutoring sessions to run smoothly.

Figura 2. A students learns how to use the AR tool.

All students were interested on learning the use of the tool, and regretted the tool was not used during the regular Differential Calculus course.

Several trials showed that the best representation was a cube around the origin of the coordinate system, with the X, Y and Z represented as lines with different colors, extending both in the positive and negative direction.

5.1. Face-to-face experience

In the face-to-face experience, both the instructor and the students met at the Virtual Reality lab at Eafit University. In the experience, each student received individual tutoring.

An informal conversation was developed with each student about the different real variable functions, their graphs and how they behaved when translated, scaled and reflected about the Cartesian plane axes. Whenever a concept was not clear, or the formulas were not remembered, the student received an explanation.

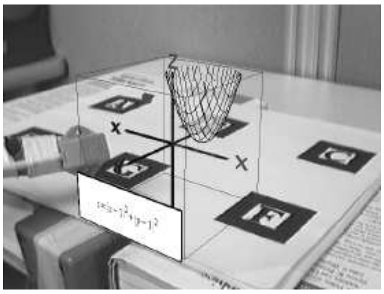
After the initial induction, each student was proposed the following question: Equations of the form $x = c$, $y = ax$ y $y = ax + b$ represent straight lines on the Cartesian plane; what type of geometric figure do they represent in the 3D space? In all cases, with no exception, students answered that these equations represented straight lines. Whenever asked for a reason, students answered that straight lines were straight lines, no matter what. When graphing the $x = 1$ with the help of the AR tool, all students were surprised. Students started moving the pattern and their heads, in order to observe the figure from different angles. Students asked to have graphics of other planes, such as $y = x + 1$ or $y = x - 1$, leading to the concept of translation of surfaces in 3D. In each case, observations were made by the instructor on the characteristics of these planes: the intersections with the axes, other forms they were parallel to. Similar reasoning took place when equations $x = 3$ and $x = -2$ were graphed.

Students were then asked about the graphics of equations of the form $y = k$, $z = k$ y $z = ax + by$. En all cases, students established relations between these new equations and the ones seen previously, determining the new characteristics of each equation.

The next equation was $z = x^2$. When asked about the form the graphic would have in 3D, several students answered that it looked like a paper sheet when the opposing edges were risen. When representing the graph using the AR tool, students were surprised to see that their intuition was correct. Students were then asked about symmetries in the surface. Then equation $y = ax^2$ was introduced. The predicted the effect of constant a on the curvature of the surface. Graphics were then showed for equations $z = y^2$ y $y = x^2$. Students were able to reach similar conclusions easily. Students were asked about the form of surfaces of the form $z = \sin(x)$, $z = \ln(x)$ and $z = e^x$, among others. In all cases, students were able to predict the correct shape, showing their ability to generalize such equations.

The experience went on with surface $z = x^2 + y^2$. In this case, students could not really infer the form of the paraboloid. Some of them assumed that the graph would grow in all directions, based on the $z = x + y$ plane. After visualizing the graph of the equation, they were able to infer the behavior of equations of the form $z = \sin(x) + \cos(x)$. They were then asked to infer the behavior of equations of the form $z = (x-a)^2 + (y-b)^2 + c$ (see figure 3). In all cases, students predicted the behavior, showing in the virtual space the approximate maximum or minimum point of the graph.

Figura 3. Graph of a translated paraboloid.



When presented with surfaces of the form $z = x^2 - y^2$, students were confused by observing that the graph grew in all directions and did not show any *regular* behavior.

5.2. Remote experience

Students in the remote experience were in the Virtual Reality lab, while the instructor was in his office, in a distant place inside the campus. The instructor and two students communicated verbally using a NetMeeting type of application. The same scheme of a review of basic concepts, as in the face-to-face experience was conducted. The experience was then conducted following the same surfaces described above. Both the instructor and the students watched the same graphs while being able to point at points in the surface.

One of the characteristics of the AR tool is that it does not allow the student to distract himself (herself), he (she) has to concentrate on following the instructor's instructions and on taking the conclusions based on his (her) observation. Real-time interaction, based on the fact that the instructor and the student were watching the same graph, was as fluid as in the face-to-face case.

This characteristic is very important for future experiences, we foresee that the experience can be repeated simultaneously with several groups of students in different geographical locations.

6. Results and evaluation

In this first experience of using the AR tool for a simulated distance-learning setting, the following aspects were found:

- No significative differences were found between the face-to-face group and the distance one.
- The AR tool allows the student to immerse in the reasoning task, making it easier for him/her to infer new

knowledge on the observed virtual objects. The interface is very natural, and does not get in the way while the student follows the instructors instructions and answers questions.

- The *micro-world* that results from the combination of virtual and virtual objects, increases the students' understanding of the task at hand.

At the end of the experience, a questionnaire was filled by the students. Some of their reflections are:

- Watching what happened with the graph was impressive. It was like having the object in my hand, and moving it as I wanted.

- The translations of the functions were a lot easier to understand this way than when they were explained to me using the Cartesian plane.

- This tool is a great help that, in the future, will become more portable and will be deployed in virtual classrooms. It is impressive to be able to use this type of tools.

- I had a better perception of what a graph can be. Before the experience, I did not imagine how a function in 3D space could be.

Finally, the two groups of students were brought together, and the differences between the face-to-face and the distance experience were analyzed. The conclusion was that the way the tool worked, and the fact that the student was immersed in the *micro-world*, the physical presence of the instructor was not actually relevant to the learning process. The real-time verbal communication and the fact that the instructor and student were sharing the same surfaces made the two learning experiences similar.

It is important to underline that in a regular multivariate calculus course, in which tools like this are not deployed, the comprehension process of these concepts is very slow. It takes between four and eight hours of explaining by the instructor. Based on the AR tool, each student comprehended the concept in about 30 to 45 minutes.

7. Future Work

We expect to further develop this tool in order to use it with large groups of students. In the presence of several students, we plan to experiment with two hardware settings: (i) provide each student with AR goggles and a paper pattern or (ii) provide each student with a TabletPC. In both cases the students' systems would be interconnected.

We also expect to use a similar tool in other science courses, such as Physics or Chemistry.

8. Conclusions

The combination of technology tools and an appropriate pedagogical support allows students to reach deeper and better understanding of the topics at hand. This type of work involves interdisciplinary groups, motivated by the common goal of allowing future generations a friendlier approach to the basic concepts in science.

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